RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION(Batch 2019-22), March 2021
SECOND YEAR

Date : 16/03/2021 **MATHEMATICS HONOURS**
Time : 11 am - 1 pm **Paper : MACT 6[CC6]** **Full Marks : 50**

Instructions to the students

- Write your **College Roll No, Year, Subject** & **Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject** & **Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer script must be numbered serially by hand.
- In the last page of your answer-script, please mention the total number of pages written so that we can verify it with that of the scanned copy of the script sent by you.
- For an easy scanning of the answer script and also for getting better image, students are advised to write the answers on single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App or any other Scanner device and make a **single PDF file(Named as your College Roll No)** and send it to [redacted]

Group - A

Answer question number 1 and any 2 questions from question numbers 2 to 4. **[15]**

1. Find the translation by which the linear terms of the equation $x^2-2y^2-3z^2-2x+4y-6z+3=0$ vanish. [3]
2. How many tangent planes may be drawn to the sphere $x^2+(y-1)^2+(z-2)^2=1$ through the line $x-1=0=y+z+1$. If there be any, find such plane or planes. [6]
3. The section of the enveloping cone of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ by the plane $y=0$ is a parabola. Find the locus of the vertex of the cone. [6]
4. Reduce the equation $3x^2-y^2-z^2+6yz-6x+6y-2z-2=0$ to its canonical form and determine its nature. [6]

Group B

Answer any 3 questions out of 5 questions in this group. **[15]**

5. Let $(a,b) \in \mathbb{R}^2$. For a fixed $\delta > 0$ the l^1 and l^2 neighborhoods of (a,b) are defined respectively as

$$U_\delta = \{(x,y) \in \mathbb{R}^2 | |x-a| + |y-b| < \delta\} \text{ and}$$
$$V_\delta = \{(x,y) \in \mathbb{R}^2 | \sqrt{(x-a)^2 + (y-b)^2} < \delta\}$$

Show that for each fixed $\delta > 0$, there exists γ and λ, both positive real numbers, such that $U_\gamma \subseteq V_\delta \subseteq U_\lambda$. [5]

6. Let $f(x,y) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right) + y^2 \sin\left(\frac{1}{y}\right) & , xy \neq 0 \\ x^3 \sin\left(\frac{1}{x}\right) & , x \neq 0, y = 0 \\ y^2 \sin\left(\frac{1}{y}\right) & , x = 0, y \neq 0 \\ 0 & , \text{ otherwise.} \end{cases}$

   (a) Are f_x and f_y continuous at (0,0) ? [3]

   (b) Check the equality of $f_{xy}(0,0)$ and $f_{yx}(0,0)$, if they exist. [2]

7. Let $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & , (x,y) \neq (0,0) \\ 0 & , \text{ otherwise.} \end{cases}$

   (a) Show that $f(x,k)$ and $f(h,y)$ are continuous for each fixed h and k. [4]

   (b) Check the continuity of f at (0,0). [1]

8. (a) Find the tangent plane to the surface $2x^2z^2 - 3xy + 4x = 7$ at the point (-1,3,1). [3]

   (b) Let $f : \mathbb{R}^3 \to \mathbb{R}$ and $\overrightarrow{F} : \mathbb{R}^3 \to \mathbb{R}^3$ have continuous first order partial derivatives everywhere. Show that [2]

$$curl(f\overrightarrow{F}) = grad(f) \times \overrightarrow{F} + f curl(\overrightarrow{F})$$

9. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be such that both the following limits exist for $(a,b) \in \mathbb{R}^2$.

$$\lim_{(x,y)\to(a,b)} f(x,y) \quad \text{and} \quad \lim_{x\to a}\lim_{y\to b} f(x,y)$$

Will these two limits be equal? Justify. (No marks will be awarded without justification) [5]

Group - C

Answer any 2 questions from question numbers 10 to 12. **[20]**

10. (a) Solve: $(x-3)\frac{d^2y}{dx^2} - (4x-9)\frac{dy}{dx} + 3(x-2)y = 0$. [5]

    (b) Solve the following system of simultaneous equations: [5]

$$4\frac{dx}{dt} + 9\frac{dy}{dt} + 2x + 31y = e^x$$
$$3\frac{dx}{dt} + 7\frac{dy}{dt} + x + 24y = 3.$$

11. (a) Solve: $\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + \frac{1}{x^4}y = \frac{2x^2+1}{x^6}$. [5]

    (b) Find the eigen-values and the corresponding eigen-functions of the boundary value problem $y'' + \lambda y = 0, y(0) + \pi y'(0) = 0, y(\pi) = 0$. [5]

12. (a) Find the power series solution near $x = 0$ of the equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + x^2y = 0$. [5]

    (b) Solve: $\frac{dx}{x^3+3xy^2} = \frac{dy}{y^3+3x^2y} = \frac{dz}{2z(x^2+y^2)}$. [5]